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A Critical Analysis of Quantitative Fingerprint Individuality Models

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ABSTRACT: A critical analysis is presented of the seven principal models previously proposed for the quantitative assessment of fingerprint individuality. Although these models have been developed over a long period of time, there has been no systematic or comprehensive review of all seven models; indeed, two very significant models have escaped the attention of contemporary investigators altogether. The present work compares and contrasts these various models. Each of the models that has been proposed is described and discussed critically in relationship to the fingerprint comparison process. None of the models previously advanced is found to incorporate the essential features of fingerprint comparison. These essential features are summarized in the context of a quantitative fingerprint individuality model.

KEYWORDS: forensic science, fingerprints, individuality, comparative analysis

Soon after the recognition of the value of fingerprints for personal identification, the degree of individuality present in a fingerprint pattern became of interest. Attempts to provide a probabilistic estimate of fingerprint individuality began with Galton's investigations in 1892 [*1*], and continue to the present day. There have been seven distinct approaches:

(1) Galton [*1*],
(2) Henry/Balthazard [*2,3*],
(3) Roxburgh [*4*],
(4) Amy [*5*],
(5) Trauring [*6*],
(6) Kingston [*7*], and
(7) Osterburg et al. [*8*].

Minor modifications of the Henry/Balthazard approach have been made by Bose [*9*], by Wentworth and Wilder [*10*], by Cummins and Midlo [*11*], and by Gupta [*12*]. Osterburg's method has been extended by Sclove [*13,14*].

Reviews and criticisms of the above approaches have been few, and none have been particularly comprehensive. Wentworth and Wilder [*15*] briefly discussed the methods of Baltha-

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zard and Galton. Roxburgh [*16*] and Pearson [*17*] have reviewed Galton's model in more detail, and Amy [*5,18,19*] has reviewed Balthazard's model. Kingston [*20,21*] reviewed the methods of Galton, Balthazard, Wentworth and Wilder, and Cummins and Midlo. Osterburg et al. [*8*] presented a critique of Kingston's method and compared Kingston's results to their own. The Federal Bureau of Investigation [*22*] has presented some brief general criticisms. The lack of a systematic, complete, and comprehensive review of existing fingerprint probability models justifies their detailed consideration here.

Galton Model, Epoch 1892

Description of the Galton Model

Galton [*1*] made the first attempt to quantify fingerprint individuality. His basic approach was to divide a fingerprint into small regions, such that the ridge detail within each region could be treated as an independent variable.

Galton worked with photographic enlargements of fingerprints. The enlargements were placed on the floor, and paper squares of various sizes were allowed to fall haphazardly on the enlarged fingerprint. Galton then attempted to reconstruct the ridge detail which was masked by the paper squares, given the surrounding ridge flow. He sought the size of square region where he could successfully predict the actual ridge detail with a frequency of one half. Galton found that for a square region "six ridge intervals" on a side he was able to predict the hidden detail correctly with a frequency of one third, and consequently concluded that a square region with five ridge intervals on a side was very nearly the size he was seeking.

To ensure that any errors would *overestimate* the chance of fingerprint duplication, Galton used a six ridge interval square region, and then assumed a probability of one half for finding the existing minutia configuration, given the surrounding ridges. The total area of a complete fingerprint was estimated to consist of twenty four such square regions. Assuming independence among these regions, Galton calculated the probability of a specific fingerprint configuration, given the surrounding ridges, $P(C/R)$, using Eq 1.

$$P(C/R) = (1/2)^{24} = 5.96 \times 10^{-8} \qquad \text{(Eq 1)}$$

Galton next estimated the chance that a particular configuration of surrounding ridges would occur. Two factors were considered: (1) the occurrence of general fingerprint pattern type, and (2) the occurrence of the correct number of ridges entering and exiting each of the 24 regions. Galton estimated the probability for coincidence of pattern type, (termed factor b by Galton), as 1/16, and the probability that the correct number of ridges would enter and exit each region, (termed factor c by Galton), as 1/256. The latter estimate was largely arbitrary, and both were presented by Galton as grossly overestimating the "true" probabilities.

Combining the frequencies of finding the necessary ridge pattern outside the six ridge interval regions with the frequencies of finding all necessary ridge detail within the regions, Galton then predicted the probability of finding any given fingerprint, $P(FP)$, using Eq 2.

$$P(FP) = (1/16)(1/256)(1/2)^{24} = 1.45 \times 10^{-11} \qquad \text{(Eq 2)}$$

Assuming a world population of approximately 16 billion human fingers, Galton concluded that, given any particular finger, the odds of finding another finger which showed the same ridge detail would be approximately one in four.

Discussion of the Galton Model

Galton's model has been criticized by Roxburgh [*16*], by Pearson [*17*], and by Kingston [*20,21*]. Most of this criticism has focused on Galton's basic assumption that, given the surrounding ridges, there is probability of one half for the occurrence of any particular ridge configuration in one of his six ridge interval regions.

Pearson considered this assumption "drastic" and suggested an alternative approach for determining the probability of a particular configuration. Assuming that the position of a minutia may be resolved to within 1 square ridge interval, there would be 36 possible minutia locations within 1 of Galton's regions. Assuming 1 minutia in each of 24 independent regions, Pearson calculated the probability of any given configuration using Eq 3.

$$P(C/R) = (1/16)(1/256)(1/16)^{24} = 1.09 \times 10^{-41} \qquad \text{(Eq 3)}$$

Pearson noted that the actual probability would be smaller for two reasons, first, because minutiae are not uniformly restricted to a single minutia in each Galton region, and secondly because of variability in minutia type.

Roxburgh's criticism of Galton's model [*4*] is more fundamental. He noted that Galton investigated only variation within single fingerprints, whereas his conclusions concerned variation among different fingerprints. This is a basic confusion by Galton of "within-group" and "between-group" variation. Roxburgh presented a series of illustrations showing that these two levels of variation need have no relationship with one another. Roxburgh conceded that Galton calculated the probability that Galton could reconstruct any particular print wholly in square regions, six ridge intervals on a side. Roxburgh argued, however, that the probability of one half for a correct guess is influenced by the size of the region relative to the ridge characteristics rather than by the variation or distribution of the characteristics themselves. If a one ridge interval square region were used an observer could always guess correctly. Under these circumstances it would be possible to reconstruct any particular print, given the ridges surrounding the squares, and yet not be able to say anything about variation between fingerprints.

Roxburgh pointed out that Galton's analysis proceeds as if he had surveyed a number of fingerprints, comparing square regions in corresponding positions within the prints. Had Galton done this, Roxburgh would have agreed with the analysis. The actual experiments, however, were quite different, and as a result Roxburgh rejected Galton's model.

Kingston [*20,21*] made somewhat the same point, noting that Galton's ability to guess the content of a square region is not an indication of the variation in actual fingerprint patterns. If Galton had shown that his region could contain only two configurations, given the surrounding patterns, Kingston would have accepted the basis for Galton's calculations. Seeing no evidence to support this contention, however, Kingston also rejected Galton's model.

In the view of the present authors, the criticisms of Kingston and Roxburgh are only partially valid. Galton intended his factors of b and c to summarize much of the variation among fingerprints. His factor b accounted for variation in general pattern type, and his factor c accounted for variation in the number of ridges entering and leaving each square region. Clearly the values of c would change radically if the size of the region were to vary. In particular, for the limiting cases where the ability to guess the content of the region approached certainty, the factor c would become very small. Unfortunately, Galton did not consider these factors in any detail, and instead chose arbitrary and excessively large estimates for both factors.

If we accept the concept of Galton's factors b and c, the question becomes whether or not Galton's experiments reasonably approximate a survey of corresponding regions in different fingerprints. It is clear that Galton had this in mind when he wrote [*23*]:

> When the reconstructed squares were wrong, they had none the less a natural appearance. . . . Being so familiar with the run of these ridges in fingerprints, I can speak with confidence on this.

My assumption is that any one of these reconstructions represents lineations that might have occurred in Nature, in association with the conditions outside the square, just as well as the lineations of the actual prints.

Galton continued, making a further assumption:

. . . when the surrounding conditions alone are taken into account, the ridges within their limits may either run in the observed way or in a different way, the chance of these two contrasted events being taken (for safety's sake) as approximately equal.

The weakness of Galton's model lies in the magnitude of the above approximation and in the arbitrary value chosen for c. We may justly criticize his final figure as a gross underestimate of fingerprint variability. Pearson's calculations of the variability in one of Galton's regions may be closer to reality, *but both his hypothesis and Galton's remain untested.*

The Henry/Balthazard Models, Epoch 1900-1943

Description of the Henry/Balthazard Models

The Henry/Balthazard approach is used in six closely related, fairly simplistic models for fingerprint individuality. Each employs a fixed probability P for the occurrence of one minutia. Assuming independence of these occurrences, the models calculate the probability of a particular configuration of N minutiae using Eq 5.

$$P(C) = (P)^N \qquad \text{(Eq 5)}$$

Henry [*2*] was the first to use this approach in 1900, and Balthazard [*3*] in 1911 made the most extensive analysis. Minor variations are encountered in the works of Bose [*9*], Wentworth and Wilder [*10*], Cummins and Midlo [*11*], and Gupta [*12*].

Henry Model (1900)—Henry [*2*] chose an arbitrary probability of one fourth for the occurrence of each minutia, as well as for the general pattern type, and the core-to-delta ridge count. To use his method, one counts the number of minutiae, and if the pattern type is visible, one adds two minutia equivalents. This value is used as N, with one fourth as P.

Balthazard Model (1911)—Balthazard's method [*3*] is particularly important because it is the historical basis for widely accepted rules regarding fingerprint individuality. Balthazard assumed that for each minutia there were four possible events:

(1) fork directed to the right,
(2) fork directed to the left,
(3) ending ridge directed to the right, and
(4) ending ridge directed to the left.

Assigning equal probability to each of these events, Balthazard took P as one fourth and N as the number of minutiae. He concluded that to observe N coincidentally corresponding minutiae it would be necessary to examine 4^N fingerprints.

Balthazard went on to calculate the number of minutiae needed for conclusive identification. His criteria was that there should be an expectation of one or less for the occurrence of the minutia configuration in the suspect population. Assuming a world population of 15 billion human fingers, 17 corresponding minutiae would be needed. (Under his model, 17 corresponding minutiae would be found with a frequency of only about 1 in 17 billion.) Balthazard considered a lesser number of corresponding minutiae (for example, 11 or 12) to be sufficient for an unequivocal identification if one could be certain that the fingerprint donor was restricted to a particular geographical area (for example, North America, California).

Bose Model (1917)—Bose [*9*] also assumed a value of one fourth for P, but he arrived at this value using a different rationale. He reasoned that there were at least four possibilities at each square ridge interval location in a fingerprint:

(1) a dot,
(2) a fork,
(3) an ending ridge, and
(4) a continuous ridge.

Wentworth and Wilder Model (1918)—Wentworth and Wilder [*10*] felt that Balthazard's value of 1/4 for P was absurdly high, and proposed a value of 1/50 for P. This represented an intuitive estimate of the value, and no evidence from which this value could be derived was presented.

Cummins and Midlo Model (1943)—Cummins and Midlo [*11*] adopted the value of 1/50 suggested by Wentworth and Wilder, but they also introduced a "pattern factor" to account for variation in the overall fingerprint pattern. Cummins and Midlo estimated that the most common fingerprint pattern occurred with a probability of 1/31. (The estimate was for an ulnar loop, and included the core-to-delta ridge count.) Equation 6 gives Cummins and Midlo's calculation for N corresponding minutiae and a corresponding pattern.

$$P(C) = (1/31)(1/50)^N \qquad \text{(Eq 6)}$$

Gupta Model (1968)—Gupta [*12*] surveyed a minutiae by first choosing a particular minutia type and position. He then searched 1000 ulnar loops for a corresponding minutia. The frequency of encountering the particular minutia type in the particular minutia position was thus determined. Gupta found that forks and ending ridges were encountered with a frequency of about 8/100, and that the remaining variety of minutia types were encountered with an average frequency of about 1/100. He therefore chose a value of $P = (1/10)$ for forks and ending ridges, and assigned a value of $P = (1/100)$ for the less common minutia types (for example, dots, hooks, and enclosures). A pattern factor of (1/10) and a factor for correspondence in ridge count (1/10) were also applied.

Discussion of the Henry/Balthazard Models

The Henry/Balthazard models share a rather casual assumption of independence, and with the benefit of statistical hindsight, most of them are arbitrary oversimplifications. Henry's method is purely arbitrary, as is Wentworth and Wilder's. Balthazard's choice of P was based on the number of possible minutia events. He has been criticized for allowing only four possible events [*24,25*] and for failing to include a "pattern factor" [*26*]. The work of Amy [*5*] has subsequently shown that Balthazard's events are not equally probable.

Bose's model does not consider the possible events for each minutia, but rather possible events at each ridge interval location. Thus, one of the allowed events is "a continuous ridge," that is, no minutia at all. Bose's assumption of equal probability for his four events is grossly in error, as pointed out by Roxburgh [*27*]. A continuous ridge is by far the most common event, and dots are much less common than either forks or ending ridges.

Gupta has an experimental basis for his minutia type frequencies, and he requires a correspondence in position as well as type. His work is weakened by his failure to define precisely the various minutia types, and by the apparently arbitrary choice of the minutia types and positions which he surveyed.

In spite of the simplicity of the Henry/Balthazard models, they may be useful as a measure of fingerprint individuality. The value of 1/4 for P may indeed grossly underestimate the individuality of fingerprints. Wentworth and Wilder's value of 1/50, or Gupta's split values of 1/10 and 1/100 could be closer to reality. In any case, there may be some empirically derived value of P for which the model is adequate. This possibility remains unexplored.

The emphasis in the Henry/Balthazard models is on variation in minutia *type*; variation as a result of minutia *location* is not explicitly considered. Most of the remaining models partition fingerprint individuality into three categories: variation in overall ridge pattern, variation in minutia location, and variation in minutia type. We will see Balthazard's two possible minutia types, with two possible orientations, incorporated into the models of Roxburgh, Amy, and Trauring. Bose's concept of minutia locations will be seen in the model of Osterburg et al. Note that Bose ignores minutia orientation, allows a wider variety of minutia types, and includes "continuous ridges" as one of the possible types.

Roxburgh Model, Epoch 1933

Description of the Roxburgh Model

The Roxburgh model [*4*] has been totally ignored by the forensic science community. Roxburgh based his model on a polar coordinate system. A configuration of concentric circles spaced one ridge interval apart is taken to represent the ridge flow of the fingerprint (see Fig. 1). An axis is drawn extending upward from the origin and is rotated clockwise. As the angle from the initial position increases, minutiae are encountered. For each minutia, the ridge count from the origin is noted, along with the type of minutia. The types of minutiae allowed by Roxburgh are identical to Balthazard's: a minutia may be either a fork or an ending ridge and may be oriented in one of two (opposite) directions. Rotation of Roxburgh's axis results in an ordered list of minutia types and ridge counts from the origin. Roxburgh decided simply to order the minutiae, rather than use an angular measure, because he could thus avoid defining his ability to resolve minutia position along each of the ridges.

After defining this system for minutia coding, Roxburgh calculated the total variability which could occur under the model. He assumed initially that the ridge count and minutia types were independent and that the possibilities for each were equally likely. Assuming n minutiae, R concentric ridges, and T minutia types, the number of possible combinations for ordered data is given by Eq 7. An additional pattern

$$\text{Number of Combinations} = (RT)^n \qquad \text{(Eq 7)}$$

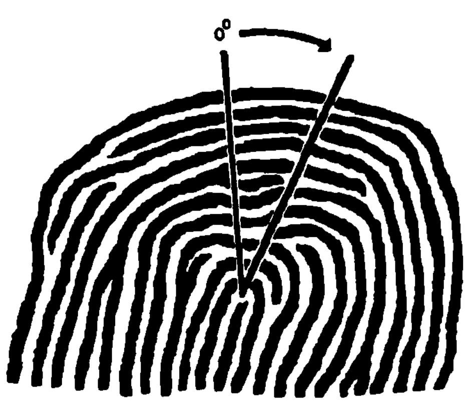

FIG. 1—*Roxburgh's polar coordinate system. The axis rotates clockwise. As minutiae are encountered, one notes the type of minutia, its orientation, and the number of ridges crossed when counting from the origin.*

factor P was introduced as an estimate for the probability of encountering the particular fingerprint pattern and core type. The total variability was calculated using Eq 8 with the following values for the parameters: $P = 1000$, $n = 35$, $R = 10$, and $T = 4$. The value of 35 for n was used because it had been

$$\text{Number of Combinations} = (P)(RT)^{n}$$
$$= 10^{3}(10 \times 4)^{35}$$
$$= 1.18 \times 10^{59} \quad \text{(Eq 8)}$$

Galton's estimate for the number of minutiae in a fingerprint, and the value of 4 for T was used because of the 4 options for minutia type. Roxburgh estimated P using Galton's fingerprint classification system wherein there are 1024 different classification types. R was taken as a conservative value; there are many more than 10 semicircular ridges in a complete fingerprint pattern. The result of Eq 8 was therefore presented as a conservative upper bound for the total number of possible fingerprint types under the model.

Roxburgh next considered the question of correlation of successive ridge counts and successive minutia types. Using 271 fingerprints, he recorded the first 4 minutiae in each print. The data were classified by the sequence of ridge numbers and the minutia types. Without statistical analysis, Roxburgh noted that there appeared to be a roughly even distribution with respect to ridge number sequences, and also with respect to the numbers of forks and ending ridges with each ridge count. He did observe, however, an excess of those minutiae which cause production of ridges compared to those which cause loss of ridges. Roxburgh attributed this to the clockwise rotation of his axis, and a general tendency for ridges to diverge as one proceeds from the vertical. The largest group of fingerprints showing the same sequence of minutia types had 8 members. Roxburgh therefore used the value of (271/8) as a conservative estimate of the variability of 4 minutiae with respect to type. For 1 minutia, the value would be the fourth root of this, or 2.412. Roxburgh proposed this corrected value of T to adjust for the observed correlation.

Up to this point, only ideal fingerprints had been considered. Roxburgh modified his model further to allow for poorly defined or poorly recorded prints. Poor recording of fingerprints may cause a true fork to appear as an ending ridge, either above or below the ridge bearing the fork (see Fig. 2). Similarly, a true ending ridge may appear as a fork, joining either the ridge above or the ridge below. It should be recognized that quite apart from recording difficulties, the nature of some minutiae may be uncertain on the skin surface itself.

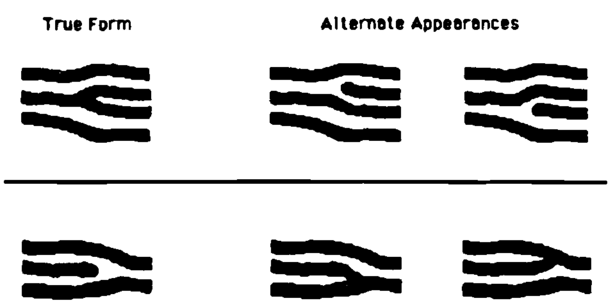

FIG. 2—*Connective ambiguities. Contingencies of fingerprint recording, deposition, and development cause variation in the appearance of minutiae of the type depicted here. These variations are referred to as connective ambiguities.*

The term "connective ambiguity" is used here to describe the general phenomenon where one is uncertain of the minutia type. In the extreme, connective ambiguity allows two additional configurations for each minutia. There is not only opportunity for change in minutia type, but for a change in ridge count as well. Roxburgh suggested the use of a factor Q to assess the contribution of connective ambiguity to his fingerprint model. The value of Q varies depending on the quality of the fingerprint. It ranges from one in an ideal print to three in a print where complete connective ambiguity must be acknowledged. (Three is used as the limit because there are three possibilities for minutia type and ridge count when complete connective ambiguity is acknowledged.) Roxburgh estimated Q as 1.5 for a "good average" print, 2.0 for a "poor average" print, and 3 for a "poor" print. The factor Q decreases the number of distinguishable minutia configurations, contributing a factor of $(1/Q)$ for each minutia. Equation 7 thus becomes Eq 9.

$$\text{Number of Configurations} = (P)[(RT)/Q]^n \qquad \text{(Eq 9)}$$

Roxburgh made one additional correction to account for circumstances where the fingerprint pattern is insufficiently clear to allow proper determination of the ridge count from the core. The relative positions of the minutiae are not affected, but there is some uncertainty about the position of the whole configuration relative to the core. A factor C was introduced, defined as the number of possible positionings for the configuration. The pattern factor P must be divided by C to correct for this uncertainty. In the extreme, where the pattern is not at all apparent, the factor P must be dropped altogether.

Roxburgh's final equation for the number of possible minutia configurations is Eq 10. For a good average fingerprint,

$$\text{Number of Configurations} = (P/C)[(RT)/Q]^n \qquad \text{(Eq 10)}$$

showing the pattern type and 35 minutiae, Roxburgh defines his variables as $T = 2.412$, $R = 10$, $n = 35$, $P = 1000$, $Q = 1.5$, and $C = 1$. Assuming each configuration is equally likely, Eq 11 gives the chance of duplication of a particular configuration of 35 minutiae.

$$\begin{aligned} P(\text{duplication}, n = 35) &= 1/(1.67 \times 10^{45}) \\ &= 5.98 \times 10^{-46} \qquad \text{(Eq 11)} \end{aligned}$$

For any particular case, Roxburgh recommends estimating the number of individuals who could have had access to the location where the fingerprint was found (be it the entire population of a country, city, or whatever). The chance of duplication of a particular configuration of minutiae in this population may then be considered, and the number of minutiae needed for any desired confidence level may be determined. Roxburgh suggested that a chance of duplication of 1 in 50 000 would be an appropriate confidence level for an identification, and presented a table with the number of corresponding minutiae needed for various populations and fingerprint qualities.

Discussion of the Roxburgh Model

Roxburgh's model is both novel and conceptually advanced. There are a number of noteworthy aspects which warrant discussion:

- the polar coordinate system,
- treatment of correlation among neighboring minutiae,

- adjustments for fingerprint quality and for connective ambiguity, and
- consideration of variation in the position of the minutia configuration relative to the pattern core.

Roxburgh introduced most of these concepts for the first time, and repeatedly drew upon actual experimental observations. His work must be considered revolutionary in these respects. It is remarkable and lamentable that Roxburgh's model has escaped the attention of subsequent investigators. No review or even a citation of Roxburgh's work appears to exist anywhere in the forensic science literature.

Roxburgh's polar coordinates are a natural choice for whorl patterns with radial symmetry, and for fingertips, where ridges are semicircular and nearly concentric. The model is not directly applicable where ridges form loops, triradii, or patternless, parallel ridges. Broader application results if the origin is allowed to move along a reference ridge. An axis may thus sweep up one side of a loop and down the other, or across a series of parallel ridges, as shown in Fig. 3.

Roxburgh briefly considered a second model, similar to Pearson's, which used rectangular coordinates to define minutia position. Each minutia was assumed to occupy 2.5 square ridge units, and minutia density was estimated as 1 per 25 units. Assuming minutiae to be evenly distributed, this allows for 10 possible positions per minutia. If each position is equally likely, then the probability for occupancy for a given minutia position is estimated at 1/10. For more accuracy, Roxburgh suggested that resolution of minutia positions be treated differently *along* ridges than *across* them. Across ridges we may easily distinguish a 1 ridge unit interval, whereas along ridges Roxburgh suggested an average resolution of 3.5 ridge intervals. Here Roxburgh points out the convenience of his polar coordinate model, in which the question of resolution need not be considered.

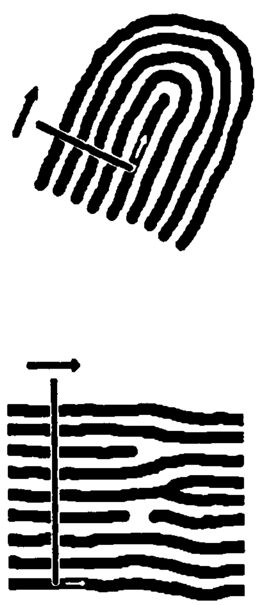

FIG. 3—*Extension of Roxburgh's method to two other ridge patterns. Roxburgh's polar coordinate model may be extended to other ridge patterns if the origin is allowed to move along a reference ridge. As examples, the origin could move up and down the core of a loop while the axis sweeps from one side to the other, or the origin could move along one ridge, while the axis sweeps across a field of parallel ridges.*

Although Roxburgh's model is simplified by avoiding minutia resolution, resolution is clearly a fundamental aspect of fingerprint individuality and its omission will be a major defect in any fingerprint model. Roxburgh argues that in practice it is relative distances between minutiae which one compares, and that the criteria for correspondence among minutia positions varies with the distance between them. Stated differently, minutiae on neighboring ridges will show comparatively less variation in relative position than will minutiae separated by several ridges. These observations help characterize the complexity of the problem, but do not diminish its fundamental importance. By sidestepping the issue of resolution, Roxburgh weakened his model.

This weakness, however, is overshadowed by the ingenuity that Roxburgh showed when he refined his model. He first considered correlation of minutiae. Even though he "eyeballed" the lack of correlation among successive ridge counts and among minutia types, his observations had an experimental basis and are distinguished as the first (and nearly only) consideration of correlation among minutiae. Roxburgh did find a correlation among minutia orientations, which he attributed to the generally observed divergence of ridges at the fingertips. A somewhat crude overcorrection was made for this correlation: Roxburgh simply assumed that the correlation for all types and orientations of minutiae was equal to the maximum that he had observed among all the various combinations.

Roxburgh next considered the effect of print quality on connective ambiguity. Galton *[28]* discussed connective ambiguity and undoubtedly made allowances for it when he judged his ability to guess ridge structures. Roxburgh, however, was the first to make specific allowance for connective ambiguity, and to link the allowance to print quality. Print quality is very important in defining how much connective ambiguity is allowable. Even in excellent prints an occasional minutia will exhibit variability in recording. The presence of more than a few would warrant suspicion of nonidentity. In poorly recorded prints, however, one must allow this variation in virtually all minutiae. In this extreme we know only that a new ridge appears in a given location. The three possible minutiae which could produce the ridge account for Roxburgh's correction factor $Q = 3$.

Roxburgh's last refinement of his model was an assessment of the uncertainty of the position of an entire minutia configuration within the overall pattern. Roxburgh observed that when one does not have a clearly defined reference point, such as a pattern core, one may make several positionings in an attempt to find a corresponding minutia configuration. The absence of a reference point thus increases the possibility of chance correspondence by a factor equal to the number of possible positionings. With hindsight, this point is obvious and amounts simply to an observation that there are several opportunities for a particular event to occur. Of the remaining fingerprint models, only those of Amy and Osterburg incorporate this important feature.

Amy Model, Epoch 1946-1948

Description of the Amy Model

Amy *[5]* defined two general contributions to fingerprint individuality: variability in minutia type (his *facteur d'alternance*), and variability in number and position of minutiae (his *facteur topologique*).

Variability in Minutia Type—Amy assumed the same possible minutia types as did both Balthazard and Roxburgh: minutiae can be either forks or ending ridges, and can have 1 of 2 (opposite) orientations. Using a data base of 100 fingerprints, Amy determined that the relative frequencies of forks and ending ridges were 0.40 and 0.60, respectively. He also noted that divergence or convergence of ridges was very common, and that when this occurs there is an excess of minutiae with 1 orientation. Amy estimated a frequency of 0.75 for minutiae with 1 orientation and a frequency of 0.25 for minutiae with the opposite orientation.

With F1 forks and E1 ending ridges in one direction, and F2 forks and E2 ending ridges in the other, Amy calculated the probability of a particular ordering (A1) using Eq 12, which reduces to Eq 13. Amy pointed out that in the general

$$P(\mathrm{A1}) = [(0.75)\,0.4]^{\mathrm{F1}}[(0.25)\,0.4]^{\mathrm{F2}}[(0.75)\,0.6]^{\mathrm{E1}}[(0.25)\,0.6]^{\mathrm{E2}} \quad \text{(Eq 12)}$$

$$P(\mathrm{A1}) = (0.3)^{\mathrm{F1}}(0.1)^{\mathrm{F2}}(0.45)^{\mathrm{E1}}(0.15)^{\mathrm{E2}} \quad \text{(Eq 13)}$$

case one does not know the absolute orientation of the minutia configuration; accordingly, a probability with the reversed orientations must be calculated. Thus $P(\mathrm{A2})$ is given by Eq 14.

$$P(\mathrm{A2}) = (0.3)^{\mathrm{F2}}(0.1)^{\mathrm{F1}}(0.45)^{\mathrm{E2}}(0.15)^{\mathrm{E1}} \quad \text{(Eq 14)}$$

The total probability of the ordering of the minutia configuration (Amy's *facteur d'alternance*) is given by $P(\mathrm{A1}) + P(\mathrm{A2})$, which reduces to Eq 15.

$$P(\mathrm{A}) = (0.1)^{\mathrm{F1+F2}}(0.15)^{\mathrm{E1+E2}}[3^{(\mathrm{F1+E1})} + 3^{(\mathrm{F2+E2})}] \quad \text{(Eq 15)}$$

Variation in Number and Position of Minutiae—Amy next considered variation in number and position of minutiae. Consider a square patch of ridges, n ridge interval units on a side. Let $P(L)$ be the probability that there will be p minutiae in the patch. Let N be the total number of arrangements of the p minutiae, and let $(N\{\mathbf{t}\})$ be the number of these arrangements which are indistinguishable from the particular arrangement of minutiae at issue. The probability of having p minutiae forming a configuration the type t in a square patch n ridge intervals on a side is defined as $P(T)$ (Amy's *Facteur Topologique*) in Eq 16.

$$P(T) = P(L\{\mathbf{n,p}\})(N\{\mathbf{t}\})/(N) \quad \text{(Eq 16)}$$

The patch size is actually variable because the borders of the patch are not precisely defined. Amy therefore summed the possible values of n, giving Eq 17.

$$P(T) = \frac{\Sigma\,[(L\{\mathbf{n,p}\})(N\{\mathbf{t}\})/N]}{\Sigma\,[P(L\{\mathbf{n,p}\})]} \quad \text{(Eq 17)}$$

Assuming a minimum distance between 2 minutiae of 1 ridge interval, there are $(n\ X\ n)$ positions in which to have p minutiae. Using an estimate of average minutia density of 1 per 22.5 square ridge intervals, the binomial theorem gives the values for $P(L)$ and N in Eqs 18 and 19. Substituting the results of Eqs 18 and 19 into Eq 17 gives Eq 20 for $P(T)$.

$$P(L) = \binom{n}{k}(0.0444)^{p}(0.9556)^{n^2-p} \quad \text{(Eq 18)}$$

$$N = \binom{n}{k} \quad \text{(Eq 19)}$$

$$P(T) = \frac{(0.9556)^{n^2}(N\{\mathbf{t}\})}{\binom{n}{k}(0.9556)^{n^2}} \quad \text{(Eq 20)}$$

It remains to calculate $N\{\mathbf{t}\}$, the total number of possible minutia arrangements which are of the particular type t (that is, indistinguishable from one given minutia configuration).

Amy noted that relative, rather than absolute, positioning is of concern, and proposed that the event necessary for positional identity between two fingerprints is only that the same number of minutiae appear on corresponding ridges of the fingerprints. This means that variation as a result of absolute positioning of minutiae along a ridge is disregarded; instead, we merely order the minutiae. Under these assumptions $N\{\mathbf{t}\}$ would include all the possible permutations of minutiae occurring on the same ridge and each of these permutations would be equivalent under the model. For one minutia on a ridge there are n possible positions, for two minutiae there are $[n(n - 1)/2]$ positions, for three minutiae there are $[n(n - l)(n - 2)/3]$, and so forth. Each ridge contributes a factor of this type to $N\{\mathbf{t}\}$, depending on the number of minutiae that appear on the ridge.

A second contribution to $N\{\mathbf{t}\}$ arises from ridges without minutiae which occur at the fingerprint border. Amy argued that there are q such minutia-free ridges at the upper border, then arrangements with $(q - 1)$ minutia-free ridges at the upper border and one minutia-free ridge at the bottom border would be indistinguishable. Generally, for q minutia-free ridges at the borders there would be $(q + 1)$ possible arrangements of these ridges, each resulting in an indistinguishable fingerprint pattern.

Based on the above two contributions, Amy calculated $N\{\mathbf{t}\}$ given n, p, and the number of ridges with 0, 1, 2, and so on minutiae. If there is only one minutia per ridge and no interior ridges without minutiae, then $q = n - p$ and $N\{\mathbf{t}\}$ is given by Eq 21. If there are z internal ridges without minutiae, Eq 22 results. Two minutiae on one ridge, and one on each

$$N\{\mathbf{t}\} = n^p(n - p + 1) \qquad \text{(Eq 21)}$$

$$N\{\mathbf{t}\} = n^p(n - p + 1 - z) \qquad \text{(Eq 22)}$$

of the others results in Eq 23. Two minutiae on each of two ridges and one on each of the others results in Eq 24, and the formula generalizes easily.

$$N\{\mathbf{t}\} = \frac{(n)(n - 1)}{2} n^{(p-2)}(n - p + 2 - z) \qquad \text{(Eq 23)}$$

$$N\{\mathbf{t}\} = \frac{(n)^2(n - 1)^2}{4} n^{(p-4)}(n - p + 3 - z) \qquad \text{(Eq 24)}$$

Correction for Minutia "Groups"—Amy noted that the foregoing theory failed when clusters of minutiae appeared on one ridge. A problem of definition results: when does one ridge become two ridges? In response to this issue Amy defined "groups." A group is an interconnected cluster of minutiae which is treated as if it were a single ridge. Amy noted that within these groups, not all relative types and positions of minutiae are possible, and some new types of positioning are created.

Consider the possible arrangements of two ridge endings shown in Fig. 4. Four possible arrangements (a through d) result because each ending has two possible orientations. If the two endings appear on the same ridge, however, the possibilities are restricted to a and b. Where patterns of multiple forks appear, there is not a loss of possible arrangements, but an increase. The possibilities for two forks are shown in Fig. 5. Four possible arrangements would be predicted, as with the ending ridges, but in fact there are eight possibilities. The multiple forks create a compound ridge wherein there is a greater potential variation.

Amy introduced a correction factor G to adjust for this deficiency in the model. G is the ratio of the number of possibilities predicted by the model to the actual number of possibilities. Thus for our two-fork example, G would be equal to one half, and for the two ending ridges, G would be two. Amy calculated G for clusters of two to six minutiae.

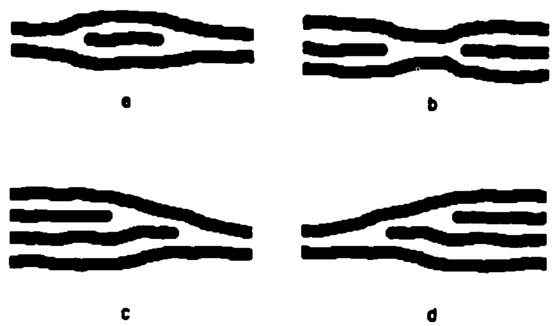

FIG. 4—*Possible configurations of two ending ridges. Configurations* a *and* b *show the endings on the same ridge location, whereas configurations* c *and* d *show the endings on two separate ridges.*

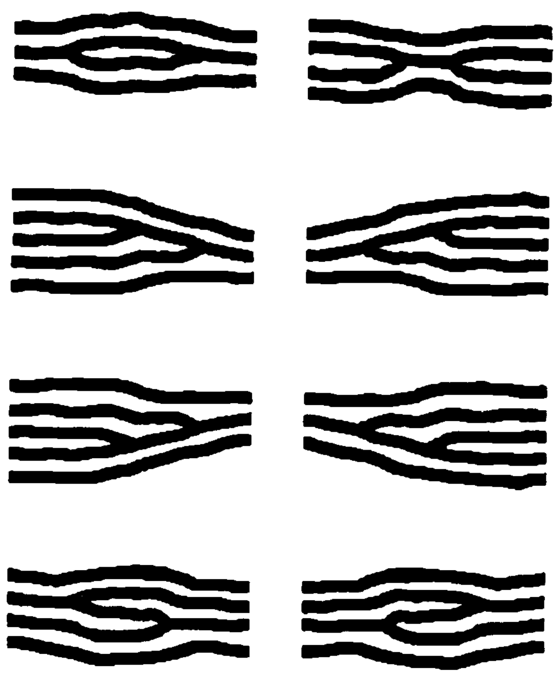

FIG. 5—*Possible configurations of two forks on one ridge. Eight configurations are possible. Only four would be predicted by the permutations of the fork orientations.*

Combined Frequency for a Minutia Configuration—Amy's estimate for the frequency of a particular minutia configuration, $P(C)$, is given in Eq 25. This incorporates the correction factor G along with $P(\mathrm{A})$ and $P(T)$ from Eqs 15 and 20.

$$P(C) = P(\mathrm{A}) \times P(T) \times (G) \tag{Eq 25}$$

Chance of False Association—Amy noted that the chance of false association depends on the number of comparisons one makes. Thus if a frequency of a particular configuration is

$P(C)$, the chance that a given area is not of this configuration is $[1 - P(C)]$, and for r comparisons the probability of association by random $P(AR)$ is given by Eq 26.

$$P(AR) = 1 - [1 - P(C)]^r \qquad \text{(Eq 26)}$$

The Taylor expansion yields Eq 27, which reduces to Eq 28 when $P(C)$ and $[r \times P(C)]$ are small.

$$P(AR) = 1 - [1 - (r)P(C) + \frac{(r)(r + 1)}{2} P(C)^2 \ldots] \qquad \text{(Eq 27)}$$

$$P(AR) = (r)\ P(C) \qquad \text{(Eq 28)}$$

Amy calculated the number of comparisons r as follows. Consider a fingerprint of unknown origin which fills a square region, n ridge intervals on a side. This print is to be compared to a larger, known fingerprint, represented by a square region with N ridge intervals on a side. The number of horizontal positions which the smaller unknown print may occupy in the larger print is given by $(N - n + 1)$. There are an equal number of vertical positions, one for each ridge. Therefore the total number of positions for comparison is given by Eq 29.

$$\text{Number of Positions} = r = (N - n + 1)^2 \qquad \text{(Eq 29)}$$

The value of N (in ridge intervals) is estimated by taking the square root of the area of the known prints. Amy estimated a value of 9.49 for thumbs, 8.37 for the other fingers, and 31.62 for palms. The total number of positions for one person is calculated in Eq 30. When palms are not examined the appropriate equation is Eq 31.

$$r = 2(9.49 - n + 1)^2 + 8(8.37 - n + 1)^2 + 2(31.62 - n + 1)^2 \qquad \text{(Eq 30)}$$

$$= 12n^2 - 256n + 1986$$

$$r \text{ (no palms)} = 10n^2 - 192n + 923 \qquad \text{(Eq 31)}$$

Areas Excluded as a Result of Pattern Elements—Up to this point only patternless fingerprints had been considered. Amy reasoned that where loops, whorls, and triradii exist, the number of positions for comparison is less, regardless of whether the unknown fingerprint contains such patterns. Assuming the worst case, with a patternless unknown print, then for each pattern present on an individual, n positions are excluded from comparison in each dimension. Letting s equal the number of pattern singularities on a person's hands, the number of comparisons may be recalculated using Eq 32.

$$\text{Number of Comparisons} = r - n^2 s \qquad \text{(Eq 32)}$$

Amy's final equation for the chance of a random association is Eq 33, which combines Eqs 25, 26, and 32.

$$P(AR) = 1 - [1 - P(A)\ P(T)(G)]^{(r - n^2 s)} \qquad \text{(Eq 33)}$$

Amy concluded by calculating the chances of random association for a series of examples with different numbers of minutiae and different group arrangements. In a subsequent paper [*18*], Amy presented tables that considerably simplify the calculation of an upper bound for the chance of random association. And in a final paper [*19*], Amy calculated the number of minutiae needed to limit the chance of random association to one in a billion. He also noted that when fingerprint files are searched as a means of developing a candidate for comparison, the factor r increases, and the criteria for identification become more stringent.

Discussion of the Amy Model

Amy's model, like Roxburgh's, has not been reviewed in the English literature. Amy himself was apparently unaware of Roxburgh's work, and possibly even of Galton's: Amy only cites the work of Balthazard.

Amy's model is comparable to Roxburgh's in complexity, innovation, and general approach. These two investigators recognized many of the same issues and their responses were understandably closely related. Both models begin by dividing fingerprint individuality into two parts: variability of minutia type and variability of minutia position.

Amy's consideration of minutia type was more sophisticated than Roxburgh's in two respects. First, Amy experimentally determined the relative frequencies of forks and ending ridges instead of assuming the two types were equally likely. Secondly, Amy made an estimate of the nonindependence of minutia orientation based on his observations in 100 fingerprints. The nonindependence occurs because ridges converge and diverge as they flow around pattern areas. Roxburgh had observed this, and corrected for it by introducing a factor based on the greatest degree of correlation he had observed. Amy's approach was more realistic because he incorporated probabilities for the alternative orientations directly into his calculation.

Amy's treatment of minutia positional variation was also more sophisticated than Roxburgh's. Amy treated both the number of minutiae and the area of the fingerprint as variables. He used the binomial theorem and an estimate of minutia density to calculate both the probability of a given number of minutiae and the probability of any particular positional arrangement. These calculations require definition of the possible minutia positions within a fingerprint. Amy assumed the minimum distance between two minutiae was one ridge interval. The number of possible minutia positions was thus equal to the area in square ridge intervals. The issue here is not the ability to resolve minutia positions, but rather to determine the number of possible minutia positions. Amy considered the problem of minutia resolution by another, more questionable, process.

When comparing fingerprints, we are unable to distinguish among all the possible minutia configurations. Roxburgh recognized this and was content to use a resolution of one ridge interval across ridges and merely to order the minutiae along ridges; this, in essence, avoids the issue. Amy's treatment was more complex, but he made a functionally equivalent approximation. Amy assumed that any positional arrangement which has the same number of minutiae on each ridge will be indistinguishable. This means that, in fact, only minutia ordering along the ridges is considered. This assumption is a serious flaw in Amy's model, even more so than in Roxburgh's. In both models, the assumption is unrealistic because our ability to distinguish minutia configurations is far greater than our ability merely to note their sequence along a ridge. In Amy's model the approximation is also particularly difficult to apply. Roxburgh only required a ridge count as a radial measure; the continuity along a particular ridge was of no concern. The Amy model strives to preserve the concept of individual ridges, while still allowing multiple minutiae on a ridge. The concept of "groups" must therefore be introduced, and interconnected ridge systems must be defined as a single compound ridge. The complexity introduced by Amy's group correction factor is awkward enough, but more importantly, Amy's model can in no way account for the connective ambiguities of minutiae. Connective ambiguities prevent the definition of discrete, interconnected ridge systems. Amy totally ignored this issue and provided no consideration of, or correction for, connective ambiguities. Inasmuch as connective ambiguities are an unavoidable feature of fingerprint comparison [*29,30*], Amy's model is consequently not a realistic assessment of fingerprint individuality.

Amy next introduced a correction factor for featureless border ridges. He stated that it does not matter whether such ridges appear above or below the central, minutia-bearing ridges: the central ridges alone contain the features which determine distinguishability among minutia configurations. This feature of Amy's model is incorrect. When a fingerprint of unknown origin is being compared to known fingerprints, any minutia-free ridges in the unknown print are very much a part of the comparison. Should the known prints have minutiae on these ridges, there is a discrepancy, and nonidentity will ensue. It is true that for the patch of ridges that Amy uses for his unknown prints we cannot distinguish among the permutations of the featureless border ridges. But when comparing these unknown prints to the larger, known prints, the permutations are by no means equivalent.

Amy concluded his work with a calculation of the chances of false association. Given the size of the ridge configuration, Amy estimated the number of possible positionings for these ridges on a person's hands. The number of positionings varies depending on the size of the ridge configuration, on whether both palms and fingers are to be considered, and on the presence of pattern elements in either the fingerprint trace or the person's hands. The purpose of calculating the possible positionings is to estimate the number of trials one has in which to find an indistinguishable ridge configuration. At each possible positioning one makes a comparison and there is a chance of false association.

Obviously, the more possible positionings for a fingerprint trace on an individual, the greater the chance of false association. This idea was scarcely novel; both Galton and Balthazard recognized one positioning for each of a person's ten fingers. Roxburgh introduced his factor C, the number of possible positionings which a configuration could have relative to the pattern core. Amy, however, extends this concept to include all the fingers and the palms. Furthermore, Amy made the critical observation that the more people to whom one compares a fingerprint, the less is the significance of any resulting association. Each person in the suspect population represents a set of trials, and each trial carries with it a chance of false association. Of all the proposed models, only Amy treats this issue properly. Others either ignore the multiple comparisons because of positionings, assume comparison with a geographically defined population, or imply that there is only one comparison. Amy alone appreciated that the number of trials defined by the suspect population is the relevant quantity.

Trauring Model, Epoch 1963

Description of the Trauring Model

Trauring [*6*] estimated the chances of coincidental fingerprint association in connection with a proposed automatic identification system. The system is based on prior selection of three reference minutiae on a finger, and the recording of a number of test minutiae. Relative coordinates derived from the reference minutiae are used to describe the positions of the test minutiae. As proposed, the test minutiae appear within the triangular region described by the reference minutiae, and the approximate positions of the reference minutiae on the finger are known.

Trauring made the following assumptions:

1. Minutiae are distributed randomly.
2. There are two minutia types: forks and ending ridges.
3. The two minutia types are equally likely to occur.
4. The two possible orientations of minutiae are equally likely to occur.
5. Minutia type, orientation, and position are independent variables.

6. For repeated registration of one individual's finger, the uncertainty in the position of the test minutiae relative to the reference minutiae does not exceed 1.5 ridge intervals.

Under these assumptions the correspondence of a test minutia requires its presence within a circular region of radius 1.5 ridge intervals (area = 7.07 square ridge intervals). The chance of a minutia appearing in this region is equal to the minutia density (s), multiplied by the area. The minutia may be one of two equally likely types, and has one of two equally likely orientations. The probability of a corresponding test minutia, given acceptable reference minutiae, P(TM/RM), is therefore given by Eq 34. If the chance of encountering an acceptable set

$$P(\mathrm{TM/RM}) = (0.707)/(4s) = 0.177(s) \qquad \text{(Eq 34)}$$

of reference minutiae on one finger is r, then each person has a $(10 \times r)$ probability that the reference minutiae will be present on one of their ten fingers. If the number of test minutiae is N, then the chance of random correspondence of any one of an individual's fingers with a previously defined fingerprint is given by Eq 35.

$$P(\text{corresponding individual}) = (10r)(0.177s)^N \qquad \text{(Eq 35)}$$

Based on his observations of 20 fingerprints, Trauring found a maximum value of 0.11 for minutia density. He also estimated that the probability of correspondence of 3 randomly corresponding reference minutiae could be conservatively taken as 1/100. Substituting these values for s and r gives Eq 36.

$$P(\text{corresponding individual}) = \frac{(0.1944)^N}{10} \qquad \text{(Eq 36)}$$

Discussion of the Trauring Model

Trauring's perspective differs somewhat from the other investigators; he developed his model not to study fingerprint individuality, but rather to estimate the ability of computerized optics to identify a particular finger. Our purpose here is to evaluate Trauring's model as it applies to fingerprint individuality, a function for which it was not actually proposed.

Automatic comparisons by computer favor the introduction of continuous rectangular coordinates. When fingerprints are compared manually, however, actual distances between minutiae are not compared; instead, ridge counts are made across ridges, and relative distances are compared along ridges. It is understandable that for computerized recording and comparison of fingerprints the ridge count might be dispensed with; in doing so, however, one departs from reality. Ridge count is an essential part of the actual identification criteria, and its omission weakens Trauring's model.

Trauring's model is similar to the Henry/Balthazard models, although better thought out. Trauring's first five assumptions are identical to Balthazard's, and the result fits the Henry/Balthazard format ($p = 0.4641$ for the three reference minutiae and thereafter $p = 0.1944$). Trauring, however, laid a better foundation for his model. His derivation was based on consideration of minutia density, estimates of error in minutia positioning, and the concept of reference and test minutiae. Trauring shares some of the faults of the simple models: he assumed minutia types and orientations to be equally probable, and considered neither connective ambiguity nor correlation among minutiae.

The most important feature of Trauring's model is his concept of reference minutiae. Trauring used the locations of three reference minutiae to bring a finger into register. Positions of the remaining "test" minutiae were determined relative to these reference minutiae.

In actual fingerprint comparison, a similar process is followed. A characteristic group of minutiae or a ridge pattern such as a loop or delta is used as a reference point, and a comparison with other prints begins by searching for this reference point. If a corresponding reference point is found, the remaining minutiae are used to test the comparison. Ridge count from the reference point, relative lateral position, orientation, and minutia type are factored into the comparison. Each minutia sought is a test of the hypothesis that the prints are from the same individual.

Although no other fingerprint model explicitly distinguishes between reference and test minutiae, the issue has arisen in different forms. Galton, Henry, Cummins and Midlo, and Roxburgh used "pattern factors" to estimate the chances of encountering a particular fingerprint pattern type. Pattern cores and delta regions, and even diverging ridges in arch patterns, provide good reference points. Roxburgh also allowed for uncertainty in position of minutiae with his factor C, the number of possible positionings of the minutia configuration relative to the pattern core. Amy considered the number of possible positionings of a fingerprint trace on the whole of the palmar surface. The "reference points" of loops, triradii, and whorls eliminate some of the possible positionings, increasing his factor n. Apart from these indirect treatments, however, the concept of test and reference minutiae remains undeveloped in its application to conventional fingerprint comparison.

Kingston Model, Epoch 1964

Description of the Kingston Model

Kingston [7] divided his model for fingerprint individuality into three probability calculations, in much the same fashion as Amy did. He first calculated the probability of finding the observed number of minutiae in a fingerprint of the observed size; next, he calculated the probability that the particular minutiae positions would be observed; and lastly, he calculated the probability that minutiae of the observed type would occupy the positions.

Probability of the Observed Number of Minutiae—Kingston estimated the probability of a particular number of minutiae from the minutia density, assuming a Poisson distribution. This assumption was justified by his experimental observations for the core area of ulnar loops. Minutia density was measured for the specific type of fingerprint pattern, and the specific location within this pattern. Graphs were constructed of expected minutia number versus the size of the sample region. For a region of given size, the expected minutia number y was read, and the probability of the observed number was calculated from Eq 37.

$$P(N \text{ minutiae}) = (e^{-y})(y^N/N!) \qquad \text{(Eq 37)}$$

Probability of the Observed Positioning of Minutiae—Kingston next calculated a probability for the observed positioning of minutiae. He assumed that each minutia occupied a square region, 0.286 mm on a side. This size region was chosen from experimental observations of minutia clustering. Within this square region, other minutiae are excluded, and thus the center positions of other minutiae are excluded from a square region 0.571 mm on a side.

An uncertainty of 0.286 mm in measurement of a minutia position was also assumed, a value based on repeated coordinate readings from a single fingerprint.

Under these assumptions Kingston proceeded as follows. Consider N minutiae occurring in a region S, square millimetres in area. The number of distinguishable minutia positions within this region is given by Eq 38. One minutia position is used for

$$(\text{Number of Positions}) = (S)/(0.571)^2 = (S)/(0.082) \qquad \text{(Eq 38)}$$

reference, and located at any position with a probability of unity. Subsequent minutiae are located with equal probability over the remaining unoccupied area. The excluded, occupied area for minutia i is given by Eq 39. The probability of the particular set of positionings is therefore calculated using Eq 40.

$$(\text{occupied area}) = (i - 1)(0.571)^2 = (i - 1)/(0.082) \qquad \text{(Eq 39)}$$

$$P(\text{positionings}) = \prod_{i=2}^{N} \frac{(0.082)}{[S - (i - 1)(0.082)]} \qquad \text{(Eq 40)}$$

The value given by Eq 40 is used except where the minutiae are sufficiently close so that the excluded regions about the minutiae overlap. Where there is overlap, the total area excluded to the next minutia is less than the given value. Presence of the overlap also causes the order in which the minutiae are taken to affect the calculation. Kingston cautioned against ignoring this overlap, but did not consider the magnitude of the error this would cause. Should significant error occur, Kingston recommended taking an average over all possible orderings of the minutiae.

Probability of the Observed Minutiae Types—To estimate the probability of correspondence in minutia type, Kingston determined the relative frequencies of minutia types in a survey of 2464 minutiae in 100 ulnar loops. The results are given in Table 1. The probability for a correspondence in minutia types was calculated as the product of the relative frequencies for each of the n minutiae.

Overall Probability of a Given Configuration—Combining the probability of the observed minutia types P with Eq 40 gives Eq 41, which calculates a value for filling the observed minutia positions with the observed minutia types. Combining Eqs 37 and 41 gives Eq 42, Kingston's final probability for a given minutia configuration $P(C)$.

$$P(\text{positions, types}) = (P_1) \prod_{i=2}^{N} (P_i) \frac{(0.082)}{[S - (i - 1)(0.082)]} \qquad \text{(Eq 41)}$$

$$P(C) = (e^{-y})(y^N/N!)(P_1) \prod_{i=2}^{N} (P_i) \frac{(0.082)}{[S - (i - 1)(0.082)]} \qquad \text{(Eq 42)}$$

Kingston included no additional factor for a correspondence in pattern type; instead, he considered the pattern singularities to be special types of minutiae. Triradii were explicitly considered as described in Table 1, and an arbitrary frequency of 0.25 was assigned to recurving ridges.

Chances of False Association—Kingston calculated the probability of false association using the Poisson distribution. Suppose there exist K persons with the given minutia configuration as seen in a fingerprint. If we select one of these persons at random, then the probability that we have the actual person who made the print is $(1/K)$. For small probabilities of

TABLE 1—*Kingston's relative frequencies of minutiae (*N *= 2464).*

Ending ridge	0.459
Fork	0.341
Dot	0.083
Enclosure	0.032
Bridge	0.019
Triradii	0.017
Other	0.031
Total	0.982 (sic)

occurrence, K takes on a Poisson distribution with parameter $y = np$, where n is the size of the relevant population and p is the probability of the event. The expectation of $(1/K)$ is thus given by Eq 43. The denominator is the Taylor expansion for the exponential function, less one. Making this substitution results in Eq 44.

$$E(1/K) = \frac{\sum\limits_{i=1} (1/K)(y^K/K!)}{\sum\limits_{i=1} (y^K/K!)} \qquad \text{(Eq 43)}$$

$$E(1/K) = 1/(e^y - 1) \sum_{i=1} (y^k)/(K)(K!) \qquad \text{(Eq 44)}$$

Whereas $(1/K)$ is the probability that the correct person is found, $1 - (1/K)$ gives the probability that an error has occurred. This value turns out to be very close to $(y/4)$ for y in the range of one to one in one billion.

Discussion of the Kingston Model

The general similarity between the Kingston model and the Amy model was mentioned earlier. Both models initially determine the probability of a particular number of minutiae in a region of given size; secondly, they compute the possible permutations of minutia positions; and thirdly, they consider variation in minutia type. The principle difference between the two models is that Amy attempts to describe minutia position within the ridge structure, whereas Kingston ignores this structure and uses the coordinates of minutia positions.

For calculating the probability of a particular number of minutiae, Kingston used the Poisson distribution; Amy had used the binomial distribution. These two probability distributions are closely related; the Poisson is merely a special case of the binomial. Both describe the distribution of the number of successes, given a number of statistically independent trials. In our specific example, *events* are the occurrence of minutiae and *the probability* is that a particular number of minutiae will occur in a region of a given size. Two parameters are necessary to describe the binomial distribution: (1) the probability of the event and (2) the number of trials. Thus, Amy used a probability of 0.0444 for a minutia to occur within a unit area and took the number of trials as total area of the region. When the number of trials is high and the probability of the event is low, the binomial probabilities are accurately given by the Poisson distribution. The Poisson distribution is described using only a single parameter: the expected number of events. Although this expectation is dependent on the number of trials and the probability of the event, these two parameters need not be individually determined. Kingston used average minutia counts in different sized regions to obtain an empirically derived Poisson parameter.

Kingston used an empirical approach because he had observed variations in minutia density among different sized regions and among different locations within the fingerprint. Amy had simply assumed a uniform minutia density. Kingston's data established that increased minutia densities occur near deltas and near loop cores. As one proceeds outward from these locations, the density falls off, creating a lower overall density as the size of the region increases. Kingston adjusted for these phenomena by restricting his consideration to circular regions about the core of loops and empirically determining the expected number of minutiae for regions of different size.

Kingston's observations of variation in minutia density are noteworthy, but his method of handling this variation within his model is open to criticism. If we accept that the density of minutiae decreases as we move outward from the core, then the probability of minutia occurrence clearly depends on the distance from the core. We may not assume, therefore, that within a circular region about the core there is a uniform probability of minutia occurrence.

Such an assumption is inherent in the use of the Poisson (or binomial) distribution, and thus there is an inconsistency in the model. We may infer that for small regions Kingston assumed that the density may be taken as constant. This assumption might well be valid in some areas of the fingerprint. Unfortunately, Kingston chose an area where the density varies dramatically. His data show that the density falls off nearly 50% as the radius of a circular region about the fingerprint core changes from three ridge intervals to six [*31*].

Kingston made this same inconsistent assumption when he considered variation in minutia position. His method was to add sequentially minutiae to a region. The probability that each successive minutia will occupy any particular position is determined by the ratio of minutia size to the remaining unoccupied space. No provision is made for the proximity of the minutia to the core, or for any variations in minutia density.

Further difficulty with Kingston's modeling of minutia position is encountered with his definitions of minutia size and resolution. Kingston assumed that each minutia occupied a square region 0.286 mm on a side. This is equivalent to 0.333 square ridge intervals. Amy had used a full square ridge interval region, thus Kingston allowed three times more minutia positions than Amy did. This difference obviously has a profound effect on number of possible minutia arrangements.

Kingston's choice of minutia size is unrealistic when evaluated within the actual ridge structure. Minutiae on adjacent ridges can be no closer than one ridge interval. Along a ridge, the question becomes one of definition. When do two minutiae which are very close become one event? Kingston did not describe his criteria for determining minutia type, but did classify "spurs" and "double bifurcations" as simple bifurcations (forks). "Dots," "enclosures," and "bridges" were given separate categories. By allowing this variety in minutia type, Kingston in effect redefined any two minutiae which appeared close to one another. Two opposing forks would be redefined as an enclosure, a fork with a quickly terminating branch would be redefined as a spur and included as a simple fork, and a very short segment of a ridge would be redefined as a dot. This redefinition of events prevents minutiae from getting closer than one or two ridge intervals from one another. The effective number of minutia positions is thus considerably less than Kingston assumed.

Kingston also used an inappropriately small value for minutia resolution. He accepted correspondence only if a minutia was found in an area of 0.082 mm^2 about the expected position. Trauring had used an area of 1.73 mm^2, even after correcting for fingerprint distortion using reference minutiae. Kingston's value was determined by noting the error in repeated measurements on a single fingerprint, in contrast to measurements of different prints from the same finger. This is a fundamental error. We are not interested in how many distinguishable patterns we may measure, but in how we may distinguish among prints from different fingers. Kingston made no allowance for the minor printing variations which are present even under ideal conditions.

Kingston's modeling of minutia positional variation, therefore, has three serious flaws: the inconsistent assumption of uniform density, excessively small minutia size, and excessively high minutia resolution. A relatively minor omission is the failure to consider positioning of the minutia configuration as a whole. Inasmuch as the Kingston model is restricted to the core areas of loops, this omission is not serious; the loop pattern allows positioning of the configuration on the finger. Furthermore, the model allows arbitrary positioning of one minutia as a starting point.

Kingston's approach to variation in minutia type differed fundamentally from the previous models. Kingston allowed a much greater variety of minutiae and assigns probabilities based on their relative frequencies. Orientation of minutiae, however, was not incorporated within the model. (It would be difficult to use orientation in a meaningful way without reference to the ridge structure.)

When minutia types other than forks and ending ridges are defined, three issues are highlighted. First, one notes that the new minutia types are compound forms of forks, ending ridges, or both. This is necessarily so, for there are only two fundamental operations that may occur to produce a new ridge. Secondly, one notes that there is a continuum between the compound forms and distinct fundamental forms. That is, for example, if two opposing forks are close to one another they are defined as an "enclosure." As the distance between the forks is increased, one finds a continuum between what is defined as an enclosure and what is defined as two distinct forks. Definition of the compound forms must therefore include a (somewhat arbitrary) judgement of where the compound character is lost. The third issue which one notes is that the frequencies of the compound forms are much lower and much more variable than those of the fundamental forms.

If one is to use the compound forms, it is appropriate to assign weights based on their frequency of occurrence. This principle has been applied subjectively for some time in fingerprint comparison [32], but Osterburg's survey [33] demonstrated that there was no consensus among fingerprint examiners regarding these frequencies. Amy [5] assigned variable weights to minutiae, but only considered the fundamental types. Santamaria [34] was the first to propose specific weighting of compound minutiae. His method was simply to assign a weight equal to the number of fundamental minutiae which were required to produce the compound one. Kingston was the first to include frequencies of compound minutiae in a model for fingerprint individuality. Gupta [12] based his model on the frequencies of compound minutia types found in specific locations within the fingerprint.

Two problems arise from Kingston's use of compound minutiae. The first, as alluded to above, is a problem of definition. When are two fundamental minutiae sufficiently close to form a compound minutia? Kingston does not state his own criteria, but he does observe that differences in minutia classification account for variation between his own frequencies and those determined by other investigators [35]. The second problem is that no provision is made for connective ambiguity. This affects not only the comparison of minutiae, but also the frequencies which are assigned. For example, a connective ambiguity at one end of an enclosure (frequency 0.032) would result in classification as either a spur, which Kingston includes with forks (0.341), or as a combination of a fork and an ending ridge ($0.459 \times 0.341 = 0.157$). The latter reclassification to two minutiae would also markedly affect the probability calculations for both the number and position of minutiae.

Kingston concluded his model with a calculation of the chances of false association, assuming a partial fingerprint with a given incidence. We have seen a variety of approaches to this problem. Galton [1] and Balthazard [3] compared the incidence of the fingerprint to the world population, and considered an identification to be absolute when the expectation within the population was less than 1. Roxburgh [4] accepted an identification when the incidence was below 1/50 000. Amy [5,19] took the actual number of comparisons into account; his chance of false association was the probability of occurrence, multiplied by the number of comparisons. Kingston's method is analogous to Galton's and Balthazard's, although his techniques are much more refined.

Using the Poisson distribution, Kingston calculated the probability that among the world population there would be N individuals with a fingerprint identical to the given one. N must be greater than or equal to one because the existence of the print is known. The Poisson probabilities are multiplied by $1/N$, which is the chance of randomly selecting any particular one of these individuals. If there is only one individual in the world with identical fingerprints, then the identification is valid; if there are two individuals, the chance is one half that the identification is valid; if three individuals, the chance is one third; and so forth.

Kingston's calculation answers the following question:

> Given a fingerprint from an unknown source, and assuming an individual is selected randomly from among all persons who actually have the fingerprint pattern on their fingers, what is the probability that this one individual made the fingerprint?

The situation is analogous to having all N persons who can make the print present in a closed room. One of the persons inside is selected randomly, and we ask for the probability that this person is the actual source of the print. We can but note that the person is one of the possible sources and that the probability is $1/N$ that we have the correct person. The frequency of the fingerprint determines the magnitude of N, the number of persons in the room.

Contrast this situation with one where the individuals within the room are selected randomly with respect to fingerprint type, and where we test the individuals to determine if they could have actually made the evidence print. The number of persons in the room N now represents a population of suspects to be tested using the evidence print. If fingerprints of an individual in this suspect group match the evidence print, what is the significance of this finding? This question parallels the practice of fingerprint comparison, whereas Kingston's does not. Kingston has assumed that his suspect has been selected solely on the basis of correspondence with the fingerprint. Rarely would this be the case; most often identification by a partial fingerprint would be a nearly independent event. The comparison would be used to test a few possible suspects rather than to define the suspect group. When many suspects are screened by means of the fingerprint, the chances of false association rise, as Amy has pointed out. Only in the hypothetically absurd extreme, where the entire population of the world is screened, is Kingston's calculation valid.

To answer the appropriate question which we posed above, one must compare the chance of the evidence occurring under two hypotheses:

H_1: that the individual in fact made the print.
H_2: that another (random) individual made the print.

Under H_1 it is certain that the print would match the individual. Under H_2 the probability is the frequency of incidence multiplied by the number of attempts we have made to compare the print. A likelihood ratio of these two probabilities gives the relative support of the evidence for the two competing hypotheses [*36*].

Osterburg Model, Epoch 1977

Description of the Osterburg Model

Osterburg, et al. [*8*] used a 1-mm grid to divide fingerprints into discrete cells. Within each cell, 1 of 13 minutia events was allowed. These events are given in Table 2 with their

TABLE 2—*Osterburg et al.'s and Sclove's relative frequencies of minutiae in 1-mm cells (*N = *8591 cells).*

Event	Osterburg et al.'s Frequency	Scove's Frequency
Empty cell	0.766	. . .
Ending ridge	0.083 2	0.497
Fork	0.038 2	0.159
Island	0.017 7	0.103
Dot	0.015 1	0.102
Broken ridge	0.013 9	. . .
Bridge	0.012 2	0.055 8
Spur	0.007 45	0.035
Enclosure	0.006 40	0.026 3
Delta	0.001 98	0.013 5
Double fork	0.001 40	0.006 37
Trifurcation	0.000 58	0.002 79
Multiple events	0.035 5	. . .
Total	1.00	1.00

observed frequencies of occurrence in 39 fingerprints (8591 cells). Under an assumption of cell independence, the probability of a given set of cell types was taken as the product of the probabilities for each cell, as given in Eq 45.

$$P\,(\text{set of cells}) = \prod_{i=1}^{n} (p_i) \qquad \text{(Eq 45)}$$

Osterburg noted that according to general practice, the weakest identification which is considered absolute is an identification based on twelve ending ridges. Equation 46 gives the probability under the model for this configuration, given an average fingerprint area of 72 mm^2. Osterburg proposed that any

$$P\,(12 \text{ endings}) = (0.766)^{60}(0.0832)^{12} = 1.25 \times 10^{-20} \qquad \text{(Eq 46)}$$

configuration of cells which exceeded this value be accepted as absolute, regardless of the actual number and type of minutiae.

Osterburg corrected for the number of possible positionings of a fingerprint in a way analogous to Amy's model. Suppose the unknown partial fingerprint occupies a rectangular region which measures W millimetres wide and L millimetres high. For a fully recorded print, these values are about 15 and 20 mm, respectively. The number of positionings of the unknown partial fingerprint on ten full fingerprints is calculated in Eq 47. The result of

$$\text{Number of Positionings} = 10(15 - W + 1)(20 - L + 1) \qquad \text{(Eq 47)}$$

Equation 45 must be multiplied by the result of Eq 47 to get the probability of random association for an individual.

The chance of false association for an individual was given using an analysis identical to that of Kingston. The essential feature is to estimate the number of possible sources K in a population of size N and weigh each probability of K by $(1/K)$.

Sclove [13, 14] made substantial modifications of the Osterburg model to account for the experimentally observed nonindependence of cells and for multiple occurrences within one cell. Sclove found that occupied cells tended to cluster. The probability that any one cell is occupied increases regularly with the number of occupied neighboring cells. To model this dependency, Sclove assumed a one-sided Markov-type process. That is, the assumption was made that the probability of a cell being occupied depends only on the outcomes of the four preceeding cells. Thus, in Fig. 6 the occupancy of cells designated X determines the depen-

X	X	X
X	Y	O
O	O	O

FIG. 6—*Sclove's one-sided Markov process for determining dependency among cells. Under Sclove's model, the conditional probability of occupancy of cell* (Y) *is determined by the occupancy of the four preceding cells* (X).

dency of the cell Y upon its neighbors. By weighting the four possible orientations of the X cells, estimates were made of the conditional probability of occupancy of Y, given the number of occupied X cells. For border cells, where information regarding the adjacent cells is incomplete, estimates of occupancy are possible using the partial information. For these cells we have a minimum and maximum number for adjacent cell occupancies.

Sclove also proposed a different treatment for multiple occurrences within one cell. Osterburg's method included a cell category of "multiple events." Based on a within-cell data analysis, Sclove justified an assumption of a Poisson distribution for the number of occurrences per cell. The mean number of occurrences in this distribution is, in turn, affected by the number of preceding occupied cells.

Sclove noted that his method avoids the need to define minutia density variations across the pattern. Local differences in density are accounted for by the dependent probabilities of cell occupancy.

Calculation of the probability for a given cell configuration proceeds by noting the number of occurrences in each cell, along with the number of preceeding occupied adjacent cells. The appropriate conditional mean is then selected from a table based on the number of occupied cells. The Poisson probability for the observed number of occurrences within each cell may then be calculated. This probability is then multiplied by the relative frequencies of any occurrences which appear in the cell. The latter were determined from Osterburg's data, and are presented here in the final column of Table 2.

Discussion of the Osterburg Model

The Osterburg model is appealing because it is simple to apply and is statistically sophisticated. It is particularly useful for the comparison of individuality among different fingerprints. If we define some standard configuration of minutiae, the model provides a means to compare other minutia configurations to the standard. The feature that allows this comparison is simply the weighting of compound minutiae by their frequencies of occurrence. Both Santamaria [*34*] and Kingston [*7*] had used this concept, but Osterburg's treatment is far more rigorous and perceptive. He has been the only investigator to consider the errors in minutia frequencies. Santamaria's method amounted to mere suggestion that compound minutiae be weighted according to the number of fundamental minutiae which compose them. Kingston used actual frequencies of occurrence of the compound minutiae, but did not consider errors in these frequencies and did not give his criteria for classification of compound minutiae types. As a result, one does not know when two closely spaced minutiae should be considered as a compound form. Osterburg defined his compound minutiae precisely, and Sclove provided definite treatment for other closely spaced minutiae.

Positioning of minutiae is also well treated for comparing the individuality of different fingerprints. Osterburg defined position using a millimetre grid which divided the fingerprint into discrete cells. Discrete cells allowed extensive treatment of correlation by Sclove, making the model robust to local variations in density. (Recall that these variations had been a major problem in Kingston's model.) Osterburg ignores positioning within the cells, but the cells are small, equivalent to about two ridges on a side. Furthermore, Sclove's treatment of multiple events provides flexibility within the cell structure.

Cells that are empty contribute to individuality within the Osterburg model. This is an important feature which has not been included in many of the fingerprint models. Bose [*9*] was the only other investigator to consider directly the value of featureless ridges. Bose's rudimentary model allowed four equally likely events at each square ridge interval, one of which was a continuous ridge. His model grossly exaggerates the value of a continuous ridge: a single ridge extending for five ridge intervals would be assigned a frequency of less than one in a thousand. It is clear, however, that a patch of ridges without minutiae does possess some individuality. Cummins and Midlo [*37*] point out that this contribution makes their estimate of fingerprint individuality more conservative. The other Henry/Balthazard models [*2,3,10,12*], along with Roxburgh [*4*] and Trauring [*6*], deny this contribution. Kingston [*7*] and Amy [*5*] indirectly address the issue. Each includes a separate calculation of the probability of finding the observed number of minutiae, given the area of the fingerprint. Amy, however, denies the value of minutia-free border ridges when he makes his final calculation of $N\{\mathbf{t}\}$. Galton [*1*] allows a factor of 0.5 for a six ridge interval square region, regardless of content. A featureless region of this size would be assigned a frequency of 0.0908 by Osterburg.

Thus far we have been discussing the use of Osterburg's model for comparing the individuality in different prints, rather than for determining the significance of a fingerprint comparison. This distinction is important. Comparison of individuality among prints amounts to determining the information content of a fingerprint pattern. In this determination we are not particularly concerned with the different ways in which the pattern may be expressed, or with the details of the pattern. Precise ridge counts would not be expected to affect the information content very much and it is reasonable that two prints differing only in the placement of one or two minutiae would have nearly the same identification value. Connective ambiguities and deformation of the fingerprint might affect the calculation of information content to some degree, but the problem is not serious. For example, a typical connective ambiguity would create uncertainty about whether a minutia was a fork or an ending ridge. We might assume the minutia to be one or the other, or perhaps take an average the two frequencies of occurrence. Deformation would affect the relationship of the fingerprint pattern to Osterburg's grid, but without gross distortion, these changes will have little effect on the overall calculation; the number of cells containing the various features would remain practically the same. Where events are grouped differently by the deformation, the effect is also small, as demonstrated by Sclove [*38*].

When Osterburg's model is used to evaluate a fingerprint comparison, however, these minor irritations become major weaknesses. The most serious weakness is the omission of the fingerprint ridge structure. Sclove [*39*] recognized this deficiency in the model, but noted that the calculations would become much more complex with a ridge-dependent metric.

Departure from the ridge structure has been discussed in connection with the Trauring and Kingston models. A model that does not recognize ridges cannot incorporate the basic features of fingerprint comparison. Relative positions of minutiae are not established by absolute distances; only in a topological sense are these positions constant. It is the ridges which serve as landmarks in fingerprint comparison by establishing relative positions through ridge count, establishing orientation of minutiae, and correcting for distortions which may be present. Trauring [*6*] at least recognized the two minutia orientations and offered a means to correct for distortions using reference minutiae. Both the Kingston and the Osterburg models ignore these fundamental issues. Osterburg's identification criteria is the occurrence of the same events in corresponding cells as defined by the grid. If a print happens to be slightly compressed or stretched, there could be no such correspondence. If deformation of the grid is allowed, then we admit that not all of the possible configurations of cells are distinguishable, and the foundation of the model is seriously threatened.

Uncertainty in positioning of the grid has a similar effect. Osterburg proposed that the grid first be placed on the fingerprint of unknown origin, and that the comparison proceed by attempting positionings on known fingerprints. Cell by cell positionings are accounted for in the model as a feature of the comparison process, but minor positionings and rotations are not. On a single print, these minor operations will create multiple descriptions under the Osterburg model. Again, this means that not all of the possible descriptions will represent distinguishable fingerprints.

The presence of a variety of descriptions within the model for a single fingerprint is suggestive of Amy's $N\{\mathbf{t}\}$, that is, the number of minutia arrangements indistinguishable from the one at issue. A correction of this type might be introduced if the number of possible descriptions for one fingerprint were calculated. The calculation would need to incorporate minor horizontal and vertical positionings, rotational positionings, and allowable deformations of the print. Some of the difficulties could be avoided if the grid were positioned in a definite manner relative to some landmark within the print. This is conceptually equivalent to using reference minutiae. The print core, delta regions, or characteristic groups of minutiae might be used. If widely spaced minutiae were used, deformation could be corrected for using Trauring's technique. If any of these modifications were made, however, the simplicity of the Osterburg model would be lost, and even though improved, the fundamental importance of ridge count would remain unrecognized.

Connective ambiguity also poses a serious problem to the use of the Osterburg model for evaluation fingerprint comparisons. For any one fingerprint there will be a variety of minutia configurations which would be identifiable. Variation in minutia type must be allowed during the comparison process. Osterburg joins the Kingston, the Amy, the Trauring, and the Henry/Balthazard models in failing to provide for this essential feature of fingerprint comparison.

Osterburg completed his model with a discussion of the probabilities of false association. Included is a correction for possible positionings, analogous to Amy's. In this respect, there is a recognition that the chance of false association increases with the number of possible comparison positions. The bulk of Osterburg's argument, however, is identical to Kingston's and suffers the same flaws. Most importantly, it assumes that an individual is selected at random from the set of persons who actually have a compatible fingerprint, rather than selected at random from a suspect population.

Conclusion: Features Sought in a Fingerprint Identification Model

Our present criticisms of existing fingerprint models have been based either on internal inconsistencies, or inconsistency with the fingerprint comparison process. The short but perceptive discussion of fingerprint models presented by the Federal Bureau of Investigation [*22*] expresses many of the same concerns. None of the models are free from conceptual flaws. Nevertheless, this review has been productive in the sense that a set of features to be included in a more comprehensive model may now be defined.

Ridge Structure and Description of Minutia Location

To bear any relation to actual fingerprint comparison, a model must be founded on the ridge structure; this is a fundamental necessity. The ridge system provides topological order to the fingerprint, correcting for minor distortions and providing the basis for comparing the relative positions of minutiae.

Locally, the ridge system defines two directions: across the ridge flow and along it. Relative positions of minutiae should be described differently along these two directions. When measuring across ridges, a discrete variable is available—the ridge count. Along the ridges a continuous linear measure should be used, and acceptable error in the continuous measure should be defined.

Description of Minutia Distribution

A description of minutia distribution is needed which incorporates these same two measures. None of the models reviewed has such a description; those models that do utilize ridge counting between minutiae require only that minutiae be ordered correctly on any given ridge.

A comprehensive description of minutia distribution must accommodate local variations in minutia density as well as variation as a result of different patterns of ridge flow. Kingston has demonstrated that rather substantial local variations in minutia density occur. These depend in part on the presence of pattern elements: recurving ridges and triradii. Sclove's modification of Osterburg's model is self-correcting for variations in density, and perhaps this model may be adapted to include discrete ridge counts.

A more fundamental relationship exists, however, between minutiae and the ridge flow. If an equal number of ridges flow in and out of a region, then for each minutia that produces a ridge there must be a minutia that consumes a ridge. Imbalance in minutia orientation results in regions with converging or diverging ridges. Thus, if the overall pattern of ridge flow is known, information is also available concerning the orientations and distribution of minutiae. This aspect of minutia distribution has not been included in any of the fingerprint models, but it is obviously an essential element of any comprehensive treatment.

Orientation of Minutiae

Along the ridge flow, two fundamentally distinct orientations of minutiae should be recognized. With the exception of the simple dot, minutiae are formed when ridges are either added to or lost from the system. Minutia orientations, like ridge counts, are robust to fingerprint distortions, and provide objective criteria for comparison.

Variation in Minutia Type

The difficulties encountered when using compound minutiae have been discussed. Minutiae are best considered as one of three fundamental types: the fork, the ending ridge, and the dot. Compound varieties arise when the fundamental types are positioned close to one another. Relative frequencies of the fundamental types should be used, but correlations with the pattern of ridge flow, neighboring minutia types, and minutia density need to be considered.

Variation Among Prints from the Same Source

Treatment of variation among fingerprints from one finger is a crucial element of a fingerprint comparison model. We have noted that ridge counts and minutia orientations are robust to fingerprint distortions. Ridge spacing, curvature, and distance between minutiae are variable. Some criteria must be given for acceptable variation in these parameters. The relevant issue is: for a particular fingerprint of unknown origin, what is the set of known fingerprint configurations which we would judge to be in agreement? We must characterize this set to calculate the chances of encountering one of its members. The problem is not simple. If an unknown fingerprint appears distorted, we would naturally tolerate more variation. We would also accept more variation as the distance between minutiae increases. A fingerprint model must incorporate these features into the comparison criteria.

Connective ambiguities must also be allowed. As noted by Roxburgh, the quality of the unknown print determines the magnitude of the allowance. In the extreme, each minutia may be considered one of three possibilities and the ridge count may be affected. It is unrealistic to allow connective ambiguity for all minutiae in the general case, but it is unremarkable to encounter a few ambiguities, even in excellent prints. The amount of ambiguity to be tolerated must be established, based on the quality of the unknown fingerprint, so that the set of acceptable known configurations may be established.

Number of Positionings and Comparisons

The value of any fingerprint for identification is inversely proportional to the chance of false association. This chance depends on the number of comparisons which are attempted.

Each attempt carries a potential for chance correspondence and the greater the number of attempts, the greater the overall chance of false association. "Attempt" means both the number of possible positionings on one individual and the number of different individuals with which the print is compared. A fingerprint model should address this issue and provide a means to determine the number of attempted comparisons.

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